Frame retrofit using friction dampers

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ABSTRACT

A simple quasi-static analysis procedure is developed for the seismic retrofit of frames using friction dampers. The procedure uses a previously published closed-form solution to establish the relationship between the response of the retrofitted frame and the damper parameters. It accounts for the effect of frame bending deflection, which may significantly influence the response estimate, by introducing a localized model to characterize a refined tri-linear force-displacement relation for the retrofitted friction damped system. The adequacy of the procedure is verified by comparing the response obtained by the proposed method with that determined from a series of dynamic analyses for a retrofitted example frame.

INTRODUCTION

The authors have recently established the closed form solution for the normalized seismic response of a friction damped system (FDS) in terms of the system parameters (Fu and Cherry 1998, 1999). This solution is based on a tri-linear structural model, to reflect the potential for damper slipping and frame member yielding. The solution was logically obtained by combining available, credible rules to permit both the equivalent linearization of the non-linear system (Iwan and Gates 1979), and the spectral response change resulting from the period shift and damping increase due to the added dissipaters (Hanson and Jeong 1994). Based on an analogy between conventional and FDSs, the force modification factor (R-factor) to be used in the design of friction damped frames (FDFs) was then defined by using the closed form solution. This approach results in a code compatible lateral force procedure for designing *new* FDFs.

The objective of the present paper is to develop a quasi-static *retrofit* analysis procedure for FDFs. Since both structural and non-structural frame damage is controlled by the frame deformation, the primary retrofit target is set to meet a specified displacement demand. The response of the FDF is determined by using the above mentioned closed form solution together with a static analysis that is based on the structural properties of the original frame. The desirable performance of the retrofitted frame can be achieved by optimizing the damper parameters in a predictable manner. In order to obtain a reliable response estimate, a localized tri-linear model is introduced to develop equations that account for the effect of frame bending deflection, which may be significant in frames that are retrofitted with friction dampers. The validity of the developed procedure is verified by comparing the response results obtained by this procedure with the corresponding results derived from a series of dynamic analyses of a retrofitted example frame.

CLOSED FORM SOLUTION FOR SINGLE DEGREE FREEDOM FDS

Fig. 1(a) illustrates the schematic model of a single degree freedom (SDOF) FDS, which consists of: a mass M, a viscous damper with coefficient C_o representing the inherent damping of the system, system gravity supporting members characterized by their lateral member stiffness K_f and yielding strength P_y , and a friction damper unit characterized by its bracing stiffness K_a and slip force P_a . The governing equation of motion of the system when excited by a ground acceleration $a_g(t)$ is:

$$\ddot{u}(t) + 2\xi_o \omega \dot{u}(t) + \frac{f(t)}{M} = -a_g(t)$$
⁽¹⁾

where u(t) is the displacement of the mass relative to ground, $\xi_0 = C_0/(2\omega M)$ is the inherent critical damping ratio of the system, $\omega = ((K_f + K_a)/M)^{1/2}$ is the system initial cyclic frequency, and f(t) is the system restoring force resulting from the resisting forces provided by the frame supporting members and the damper unit. The FDSs discussed in this paper exhibit the tri-linear f(t)-u(t) relation shown in Fig. 1(b). In Fig. 1(b), u_s and u_y denote the system displacement when the damper slips and the supporting members yield, respectively. $f_y = P_s + P_y$ denotes the ultimate strength of the system, whereas $f_s = P_s + K_f u_s$ defines the force level above which the damper is triggered to dissipate energy.

The model shown in Fig. 1(a) is composed of an original system, having a mass, viscous damper and supporting members,

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and a friction damper unit. For a given earthquake, the response of the original linear system (P_y large enough to avoid member yielding) only depends on K_f and C_o of the original system. A change in supporting member strength P, and the addition of a friction damper unit with various parameters (K_a and P_s) will alter the system response. By employing a 'resolution-synthesis' approach (Fu and Cherry 1998, 1999), the maximum seismic response of the tri-linear FDSs can be evaluated as the product of the response of the linear original system and a normalized response function which accounts for the response change due to the attached damper and the yielding of the supporting members. The response evaluation equations are presented here as Eq. 2; the detailed deduction of these parameters is given by Fu and Cherry (1999).

$$u_{\max} = R_{sd} u_{\max,o} , \qquad f_{\max} = R_f f_{\max,o}$$
(2)

where u_{max} and f_{max} are the maximum displacement and force, respectively, of the friction damped system, $u_{max,o}$ and $f_{max,o}$ are the maximum displacement and force, respectively, of the original elastic system, and R_{sd} and R_f are the normalized displacement and normalized force functions, respectively. The R_{sd} and R_f functions are calculated from:

$$R_{sd} = \sqrt{[(1 - e^{-B\xi_e})\xi_o]/[(1 - e^{-B\xi_o})\xi_e]} K_{eo}^{-\frac{3}{4}}, \qquad R_f = R_{sd}(\alpha_a / \mu_s + 1 / \mu_y)$$
(3)

where B=30 is a constant that reflects the effectiveness of damping on response reduction (Hanson and Jeong1994). The equivalent damping ratio ξ_e and the intermediate parameters E_{do} and K_{eo} are defined as:

$$\xi_{e} = \xi_{o} + E_{do} / (\pi K_{eo}), \quad K_{eo} = \alpha_{a} (\ln \mu_{s} + 1) / \mu_{s} + (\ln \mu_{y} + 1) / \mu_{y}, \quad E_{do} = \alpha_{a} (\mu_{s} - 1)^{2} / \mu_{s}^{3} + (\mu_{y} - 1)^{2} / \mu_{y}^{3}$$
(4)

In the above equations, α_a is the damper bracing stiffness ratio, μ_s is the damper slip ratio and μ_y is the system ductility ratio. These parameters are defined as follows:

$$\alpha_a = K_a / K_f$$
, $\mu_s = u_{max} / u_s \ge 1$, $\mu_y = u_{max} / u_y \ge 1$ (5)

 α_a , μ_s and μ_y are the basic system parameters. They determine the normalized response functions and therefore influence the maximum system response.

INFLUENCE OF FRAME BENDING DEFLECTION

The total lateral seismic deflection of a frame can be separated into its shear and bending deflections. The frame shear deflection is determined by the flexural properties of the frame members, whereas the frame bending deflection depends mainly on the axial deformations of its columns. These two types of deflection are independent. When a supplemental damper is added to the frame, it only changes the shear resistance capacity of the frame. The localized tri-linear model shown in Fig. 2 can be used to account for the effect of frame bending deflection in a FDS. In Fig. 2, the deformation u(t) of the mass M is the summation of the deformations of the frame shear and bending components. The frame bending component has a stiffness K_{fb} , whereas the frame shear component has a stiffness K_{fs} and a yield strength P_y . In the localized tri-linear model, the added damper unit, whose stiffness and slip force are K_a and P_s , respectively, acts in parallel with the frame shear component. This localized model was successfully used by Kasai et al. (1998) to analyze a non-yielding, bi-linear hysteretic system. The relation between system force and system displacement (f(t)-u(t)) of the localized model remains tri-linear. Hence, the equations for system response evaluation (Eqs. 2 to 5) remain valid and can be evaluated once the expressions for the basic system parameters (Eq. 5) can be established.

For the system shear component, the three local basic parameters corresponding to Eq. 5 can be defined as:

$$\alpha'_{a} = K_{a} / K_{fs}, \quad \mu'_{s} = u'_{max} / u'_{s} \ge 1, \quad \mu'_{y} = u'_{max} / u'_{y} \ge 1$$
 (6)

where u'_{max} , u'_s and u'_y are the maximum, the slip and the yield displacements, respectively, of the shear component. It is noted that the stiffness of the localized original system (system without the damper unit) is the same as the tangent stiffness of the localized system when the damper slips but the frame does not yield. Thus, the total system member stiffness and the damper bracing stiffness ratio, which can be obtained from their basic definitions (see Fig. 1(b)), are:

$$K_{f} = K_{fs} / (1 + K_{fs} / K_{fb}), \qquad \alpha_{a} = \alpha'_{a} / [1 + (1 + \alpha'_{a})(K_{fs} / K_{fb})]$$
(7)

Applying the conditions of force equilibrium and displacement compatibility to the localized system when the damper slips and the frame yields, the system slip and ductility ratios can be expressed in terms of the local parameters as:

$$\mu_{s} = \frac{\mu_{s}' + (\mu_{s}' / \mu_{y}' + \alpha_{a}')K_{fs} / K_{fb}}{1 + (1 + \alpha_{a}')K_{fs} / K_{fb}} , \qquad \mu_{y} = 1 + \frac{\mu_{y}' - 1}{1 + (1 + \alpha_{a}' \mu_{y}' / \mu_{s}')K_{fs} / K_{fb}}$$
(8)

The basic local parameters defined in Eq. 6 are physically more meaningful than the global system parameters expressed in Eq. 5, since the former can be assigned values for design while the latter are only intermediate variables used in the closed-

form response estimates. From Eq. 7, it can be seen that the introduction of frame bending stiffness $(K_{fs}/K_{fb}\neq 0)$ leads to a global α_a that is less than the local α'_a ; Eq. 3 illustrates that this results in a reduction of the damper effectiveness (Fu and Cherry 1999). Eq. 8 indicates that the global system slip and ductility ratios are greater than their local counterparts.

In the above equations, the ratio of K_{fs}/K_{fb} is a measure of the significance of the frame bending effect. If K_{fb} is much larger than K_{fs} , the frame bending effect can be neglected. However, it is difficult to estimate directly the K_{fs}/K_{fb} ratio for a frame. It may be more convenient to calculate this ratio in terms of the ratio of the period T_f of the original system ($K_a = 0$), to the period T_{fs} of the original shear system ($K_a = 0$, $K_{fb} = \infty$). Thus:

$$K_{fs} / K_{fb} = (T_f / T_{fs})^2 - 1$$
(9)

REPRESENTATIVE RESPONSE OF MULTI DEGREE OF FREEDOM FDF

When an earthquake excited multi degree of freedom (MDOF) FDF deforms in one predominant mode $\{\phi\}$, it is possible to condense the governing equations of motion of the MDOF system to a SDOF equation by introducing the coordinate transformation $\{u(t)\}=\{\phi\}q(t)$. In this transformation, $\{u(t)\}$ is the floor displacement vector of the MDOF system relative to its moving base and q(t) is a generalized coordinate. Under this condition, the condensed equation can be expressed as:

$$\ddot{q}(t) + 2\xi_{o}\omega\dot{q}(t) + \frac{f_{c}(t)}{\{\phi\}^{T}[M]\{\phi\}} = -\gamma_{c}a_{g}(t) , \quad \gamma_{c} = \frac{\{\phi\}^{T}[M]\{l\}}{\{\phi\}^{T}[M]\{\phi\}} , \quad f_{c}(t) = \{\phi\}^{T}\{f(t)\}$$
(10)

where ξ_o and ω are the inherent damping ratio and the initial frequency, respectively, of the frame, [M] is the frame mass matrix, {f(t)} is the system restoring force vector resulting from the resisting forces provided by the frame members and damper units, and γ_c is the modal participation factor. It may be seen that the relation between the condensed force $f_c(t)$ and the generalized displacement q(t) of a MDOF FDF (Eq. 10) is the same as the relation between f(t) and u(t) for the SDOF FDSs described by Eq. 1. Thus, all concepts and equations developed previously (Eqs. 2 to 9) for SDOF FDSs can be applied, by analogy, to the condensed MDOF FDF.

As illustrated by Fu and Cherry (1998, 1999), it is possible to constrain a FDF to develop a linear mode shape, $\{\phi\}$; this ensures an equal drift ratio for all storeys of the frame. For simplicity, this deformed shape $\{\phi\}$ can be set equal to $\{h\}$, the storey height vector measured from the frame base to each storey floor level. Then the q(t) and f_c(t) in Eq. 10 physically represent the angle of the deformed shape and the base overturning moment, respectively, of the FDF. i.e.:

$$\{u(t)\} = \{h\}q(t), \qquad f_c(t) = \{h\}^1\{f(t)\}$$
(11)

In summary, the seismic response of a MDOF FDF can be approximately represented by the deformed angle (DA) and base overturning moment (OTM) of the frame. These response parameters for the damped frame can be determined from the corresponding response parameters of the original undamped frame by means of the normalized functions shown in Eq. 2, in which u_{max} is replaced by DA and f_{max} by OTM. The normalized functions are evaluated from the basic local parameters (α'_a , μ'_s , μ'_y and K_{fs}/K_{fb}) through Eqs. 8, 7, 4 and 3. For the original undamped frame, the OTM can be directly established from the given lateral forces which result in the maximum displacement shape {u(t)}. The DA can then be evaluated by approximating the actual deformed shape by a straight line using the least-square-fit equation, which yields:

$$DA = \{h\}^{T} \{u(t)\} / \{h\}^{T} \{h\}$$
(12)

PARAMETERS FOR RETROFIT ANALYSIS

There are four parameters (α'_a , μ'_s , μ'_y and K_{fs}/K_{fb}) that influence the response of the retrofitted frame. The K_{fs}/K_{fb} ratio is a property of the original frame; it can be calculated from Eq. 9. μ'_y , which is defined as the ratio of u'_{max} to u'_y (Eq. 6), is not an independent design parameter, since the yield displacement of the shear component u'_y is a property of the original frame. From its definition, μ'_y can be iteratively evaluated from the following expression:

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$$\mu'_{v} = u_{\max} / u_{vo} [1 + K_{fs} / K_{fb} (1 - \alpha_{a} / \mu_{s} - 1 / \mu_{v})] \ge 1$$
(13)

where u_{yo} is the yield displacement (in terms of the DA) of the original frame, which can be obtained from a static push over analysis. Thus, the only two remaining parameters, α'_a and μ'_s , can be independently assigned values to achieve the desired retrofit results. Parameter α'_a is used to specify the damper bracing stiffness, and parameter μ'_s is used to specify the shear displacement at which the friction damper slips. In practice, it is more convenient to specify the damper slip force P_s rather than μ'_s . In order to do so, Fu and Cherry (1999) have derived the following set of equations to establish the relationship between the global system parameters and the physical parameters of the frame and damper units:

$$\frac{f_{\text{max}}}{f_{s}} = 1 + \frac{1}{1 + \alpha_{a}} (\frac{\mu_{s}}{\mu_{y}} - 1) , \quad f_{\text{max}} = \{h\}^{T} \{F\} , \quad f_{s} = \Sigma (1 + \frac{K_{a,i}}{K_{f,i}}) H_{i} P_{s,i} , \quad \alpha'_{a} = \frac{\Sigma K_{a,i} H_{i}^{2}}{\Sigma K_{f,i} H_{i}^{2}}$$
(14)

where f_{max} is the maximum system condensed force (see Eq. 2), {F} is the lateral force vector that describes the maximum frame forces developed in the retrofitted structure, and $P_{s,i}$, $K_{a,i}$, $K_{f,i}$, and H_i respectively denote the damper slip force, damper bracing stiffness, storey shear stiffness, and storey height at the i-th storey level of the frame. By imposing conditions that ensure the frame develops a linear deformed shape under the lateral force pattern (i) before any friction damper slippage occurs and (ii) when the system force reaches its maximum magnitude, the damper parameters at each storey level can be determined from (Fu and Cherry 1999):

$$K_{a,i} = K_{f,i} \frac{V_i H_i \Sigma(K_{f,i} H_i^2)}{K_{f,i} H_i^2 \Sigma(V_i H_i)} (1 + \alpha'_a) - 1, \quad P_{s,i} = V_i - K_{f,i} H_i \frac{\Sigma[(1 + K_{f,i} / K_{a,i})V_i H_i] - f_s}{\Sigma[(1 + K_{f,i} / K_{a,i})K_{f,i} H_i^2]}$$
(15)

where V_i is the maximum storey shear force at the i-th storey level of the frame, which is calculated from $\{F\}$.

FRAME RETROFIT EXAMPLE

A single bay 6-storey steel moment resisting frame (MRF) was designed in accordance with the National Building Code of Canada (NBCC 1995) for an office building location in downtown Vancouver. Details of the frame parameters and properties are provided in Table 1. The design of the MRF was governed by the code storey drift limitation of 0.02 for an elastic base shear force of 3922kN. In this paper, the MRF is used as an original frame to illustrate the proposed retrofit procedure using friction dampers. The target of the retrofit is to limit the storey drift ratio to less than 0.01, and to minimize the resulting increase in the system force.

Based on the assumption of a single predominant mode response, and applying the format of the NBCC (1995) equations to establish an equivalent pseudo acceleration spectrum, the elastic base shear force, V_{eo} , for the original elastic frame can be determined from the condensed equation (Eq. 10) as:

$$V_{eo} = L_{M}S_{pa}W, \qquad L_{M} = \frac{(\{1\}^{T}[M]\{\phi\})^{2}g}{(\{\phi\}^{T}[M]\{\phi\})W} , \qquad S_{pa} = \frac{U}{R}\frac{15vIF}{\sqrt{T}} = \frac{0.36}{\sqrt{T}}$$
(16)

where L_M is the effective modal mass factor, which in practice can be assigned a value of unity to retain conservatism, g is the gravitational acceleration, W is the total weight, S_{pa} is the representative of the pseudo acceleration spectrum for 5% critical damping ratio, which is deduced from the NBCC (1995) seismic base shear force equation, U=0.6 is a calibration factor, R=1 is the force modification factor, v=0.4 (0.2 for original design) is the zonal velocity ratio, I=1 is the importance factor, and F=1 is the foundation factor.

For the original MRF, whose period (T_f) is 2.08 second, Eq. 16 yields $S_{pa}=0.25g$, $L_M = 0.78$, and $V_{eo} = 3930$ kN. The representative response (DA and OTM) of the original elastic frame can be obtained from the static displacement { u_{eo} } and the lateral force { F_{eo} } corresponding to this base shear force. Thus, from Eqs. 12 and 11, the deformed angle DA_o= 0.0172 and the base overturning moment OTM_o=62227 kN-m. DA_o, by definition, is a measure of average storey drift of the original frame; it significantly exceeds the specified requirement under the given seismic load. From Eq. 9, $K_{fs}/K_{fb}=0.16$, based on the shear period $T_{fs}=1.93$ seconds, which corresponds to the original frame whose column axial deformations are restricted. The yield displacement of the original frame, u_{yo} (in terms of DA_{yo}=0.0075) is obtained from the static pushover curve of the original frame. Once these parameters are established, the representative response of the FDF can be expressed in terms of α'_a and μ'_s , as shown in Fig. 3.

In Fig. 3, the solid thick lines denote the response for the inelastic frame, while the dashed lines denote the response for the elastic frame. The intersection of these lines defines the onset of yielding in the retrofitted frame; a frame will only yield if its response is located to the right of the indicated yield line. It can be seen that an increase in bracing stiffness (large α'_a) leads to a reduction in the displacement, while an increase in the damper slippage (large μ'_s) can effectively reduce the system force. However, stiff braces imply correspondingly large member cross section areas, while extensive slippage can result in excessive displacements. Given the practical limitations imposed by the economic considerations of bracing member size, and by the requirement of satisfying a specified deformed angle, a value of $\alpha'_a = 4$ is selected as a reasonable

effective retrofit parameter. Similarly, $\mu'_s = 5$ is selected to limit the increase in system force, in order to avoid frame yielding.

For these selected values of α'_a and μ'_s , the corresponding estimates for the response of the FDF are DA=0.0072 and OTM=42118 kN-m. Since the DA is a measure of the average storey drift, this DA value will likely assure the satisfaction of the specified drift requirement. The OTM, which is the maximum system force f_{max} appearing in Eq. 14, is used to calculate the required system slip force (f_s) and to determine the frame storey maximum shear forces (V_i) from the same lateral force distribution pattern that was utilized for the original frame ({Feo}). Then the required bracing stiffness ($K_{s,i}$) and slip force ($P_{s,i}$) for the friction dampers are determined by using Eq. 15. Chevron braces, whose cross sections are sized according to $K_{s,i}$, were employed to accommodate the friction damping mechanism in this retrofit design example. The desired physical parameters of friction damper units are listed in Table 1.

COMPARISON BETWEEN STATIC ESTIMATE AND DYNAMIC RESPONSE

Following the dynamic procedure proposed by FEMA-273 (1997), the acceleration-time histories of ten recorded earthquakes were adjusted in the manner described by Naumoski's (1985), to ensure that their resulting 5% damped pseudo acceleration spectra matched the NBCC (1995) design spectrum described by Eq. 16. The ten selected earthquakes were El Centro (1940 NS), Taft (1952 EW), Hachinohe (1968 EW), Parkfield (1966 CSA2 N65E), Mexico (1985 SCT EW), Sylmar (1994 Northridge CHPL NS), Pacoima (1971 San Fernando N164E), Newhall (1994 Northridge LA CFS NS), Olympia (1965 N266E), and Loma Prieta (1989 Gilroy 2SHCB NS). These spectrum-compatible time-histories were subsequently used in the dynamic analyses of the original and retrofitted frames.

The envelopes of maximum storey drift developed in the retrofitted and original frames by the ten individual earthquakes are shown in Fig. 4, together with an estimate of the response obtained using the quasi-static procedure proposed in this paper. It can be seen that, except for two individual cases, the storey drifts of the retrofitted frame are controlled within the specified 0.01 limit. As well, in comparison with the original frame, the retrofitted frame exhibits an improved drift distribution. The thick dashed line labeled "static estimate: original MRF" is determined from the static response { u_{eo} } of the original elastic frame under the code lateral forces { F_{eo} } (see Eq. 16). The thick solid line labeled "retrofitted frame: static estimate" is derived by scaling the static estimate of the original MRF by the normalized function R_{sd} , corresponding to the selected system parameters (Eq. 3). The static estimate of the retrofitted frame appears to represent the average of the dynamic responses. The dynamic analysis results indicate that no major yielding occurred in the retrofitted frame members.

CONCLUSIONS

1. A simple quasi-static analysis procedure has been developed for the seismic retrofit of frames using friction dampers. The procedure employs the authors' previously developed closed-form solution for the seismic response of tri-linear hysteretic systems. This solution statically predicts the response of a retrofitted frame in terms of the original frame response and the damper parameters. The approach provides a clear description of the effectiveness of the friction dampers in improving system performance and in illustrating the relation between the system response and the damper parameters of the retrofitted frame. It is therefore more informative and efficient than dynamic analysis procedures.

2. The above mentioned closed form solution is extended to account for the effect of frame bending deflection on the system response. The extension introduces a localized model to characterize a refined tri-linear force-displacement relation of the friction damped retrofitted system. It is concluded that if the frame shear stiffness is larger than about 5% of the frame bending stiffness, the frame bending stiffness cannot be neglected when estimating the response of the retrofitted frame. Small frame bending stiffness results in a decrease in the global damper bracing stiffness ratio, which leads to a decrease in the effectiveness of friction dampers in reducing seismic response. The influence of this effect is more pronounced when stiff damper braces are employed.

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Table 1 Example frame parameters and properties

Table. T Example frame parameters and properties							
		Original frame			Damper units		
Storey level	Mass (ton)	Beam section (W)	Column section (W)	Shear stiffness K _{fs,i} (kN-m)	Brace section (HSS)	Slip force P _{s,i} (kN)	Stiffness ratio $\alpha_{a,i}$
6 5	320 346	610x140 610x140	360x314 360x314	32474 41382	152x152x6.4 152x152x11	189 587	3.3 4.3
4	346	610x262	360x382	54447	203x203x9.5	801	3.9
3	346	610x262	360x382	55471	203x203x13	1115	5.0
2	346	610x262	360x421	60615	203x203x13	1247	4.5
1	346	610x262	360x421	91435	203x203x13	1247	3.0

Note: (1) Geometry: frame span 5m, storey height 3.6m. Chevron brace for damper units. (2) Storey gravity loads: total 329kN except 285kN at top; beam 45kN except 39kN at top. (3) Material: W - wide flange steel, HSS - hollow section steel. Strength 300MPa. (4) Period of the original frame: $T_f=2.08$ sec.; (shear) $T_{fs}=1.93$ sec. (5) Elastic base shear for the original MRF design: 3922kN. (6) Response of the original frame at initiation of yielding: DA_{yo}=0.0074, OTM_{yo}=27062 kN-m. (7) Frame shear stiffness and damper slip force are measured in the horizontal direction.















Figure 4. Envelopes of maximum storey drift